HEX-Programs with Existential Quantification

Thomas Eiter, Michael Fink, Thomas Krennwallner, Christoph Redl

{eiter,fink,tkren,redl}@kr.tuwien.ac.at

TECHNISCHE
UNIVERSITÄT
WIEN
Vienna University of Technology

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Motivation

HEX-Programs

- Extend ASP by external sources
- Traditional safety not sufficient due to value invention
- Notion of liberal domain-expansion safety guarantees finite groundability

Example

$$\Pi = \left\{ \begin{array}{ll} r_1 : t(a). & r_3 : s(Y) \leftarrow t(X), \&cat[X, a](Y). \\ r_2 : dom(aa). & r_4 : t(X) \leftarrow s(X), dom(X). \end{array} \right\}$$

Contribution

- Domain-specific existential quantification in rule heads
- Grounding algorithm extended by application-specific termination hooks
- Instances: model computation over acyclic programs, query answering over programs with logical existential quantifier, function symbols

HEX-Programs

HEX-programs extend ordinary ASP programs by external sources

Definition (HEX-programs)

A HEX-program consists of rules of form

$$a_1 \vee \cdots \vee a_n \leftarrow b_1, \ldots, b_m, \text{not } b_{m+1}, \ldots, \text{not } b_n,$$

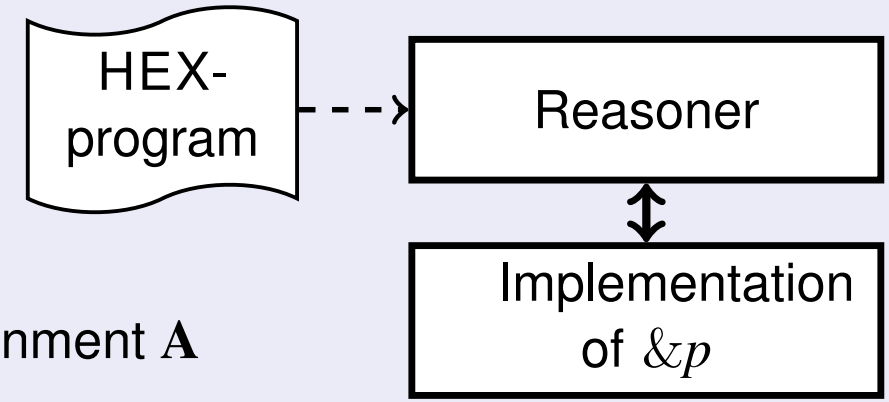
with classical literals a_i, and classical literals or an external atoms b_j.

Definition (External Atoms)

An external atom is of the form

$$\&p[q_1, \ldots, q_k](t_1, \ldots, t_l),$$

p ... external predicate name
q_i ... predicate names or constants
t_j ... terms



Semantics:
$1 + k + l$-ary Boolean oracle function $f_{\&p}$:
$\&p[q_1, \ldots, q_k](t_1, \ldots, t_l)$ is true under assignment $\mathbf{A}$
iff $f_{\&p}(\mathbf{A}, q_1, \ldots, q_k, t_1, \ldots, t_l) = 1$.

Domain-specific Existential Quantification

Idea

- Introduce new values which may appear in answer sets
- Structure of these values matters
- Introduction may be subject to constraints outside the program

Realization: Use value invention in rule body, transfer new values to the head

Example

$$iban(B, I) \leftarrow country(B, C), bank(B, N), \&iban[C, N, B](I).$$

Example

$$lifetime(M, L) \leftarrow machine(M, C), \&lifetime[M, C](L).$$

Existential Quantification

We will now discuss 3 instances of our approach:

- Model-building over acyclic $\text{HEX}^{\exists}$-Programs
- Query Answering over positive $\text{HEX}^{\exists}$-Programs
- Function Symbols

Algorithm BGroundHEX

Input: A HEX-program Π
Output: A ground HEX-program Π_g

$\Pi_p = \Pi \cup \{r_{inp}^{\&g[\vec{Y}](\vec{X})} \mid \&g[\vec{Y}](\vec{X}) \text{ in } r \in \Pi\}$

Replace all external atoms $\&g[\vec{Y}](\vec{X})$ in all rules r in Π_p by $e_{r,\&g\vec{Y}}(\vec{X})$

while *Repeat()* **do**
- $PIT \leftarrow \emptyset$
- $NewInputTuples \leftarrow \emptyset$
- **repeat**
  - $\Pi_{pg} \leftarrow \text{GroundASP}(\Pi_p)$
  - **for** $\&g[\vec{Y}](\vec{X})$ *in a rule* $r \in \Pi$ **do**
    - $\mathbf{A}_{ma} = \{\mathbf{T}p(\vec{c}) \mid a(\vec{c}) \in A(\Pi_{pg}), p \in \vec{Y}_m\} \cup \{\mathbf{F}p(\vec{c}) \mid a(\vec{c}) \in A(\Pi_{pg}), p \in \vec{Y}_a\}$
    - **for** $\mathbf{A}_{nm} \subseteq \{\mathbf{T}p(\vec{c}), \mathbf{F}p(\vec{c}) \mid p(\vec{c}) \in A(\Pi_{pg}), p \in \vec{Y}_n\}$ *s.t.* $\nexists a : \mathbf{T}a, \mathbf{F}a \in \mathbf{A}_{nm}$ **do**
      - $\mathbf{A} = (\mathbf{A}_{ma} \cup \mathbf{A}_{nm} \cup \{\mathbf{T}a \mid a \leftarrow\in \Pi_{pg}\}) \setminus \{\mathbf{F}a \mid a \leftarrow\in \Pi_{pg}\}$
      - **for** $\vec{y} \in \{\vec{c} \mid r_{inp}^{\&g[\vec{Y}](\vec{X})}(\vec{c}) \in A(\Pi_{pg}) s.t. \mathit{Evaluate}(r_{inp}^{\&g[\vec{Y}](\vec{X})}(\vec{c})) = \mathit{true}\}$ **do**
        - Let $O = \{\vec{x} \mid f_{\&g}(\mathbf{A}, \vec{y}, \vec{x}) = 1\}$
        - $\Pi_p \leftarrow \Pi_p \cup \{e_{r,\&g[\vec{y}]}(\vec{x}) \vee ne_{r,\&g[\vec{y}]}(\vec{x}) \leftarrow \mid \vec{x} \in O\}$
        - $NewInputTuples \leftarrow NewInputTuples \cup \{r_{inp}^{\&g[\vec{Y}](\vec{X})}(\vec{y})\}$
  - $PIT \leftarrow PIT \cup NewInputTuples$
- **until** Π_{pg} *did not change*

Remove input auxiliary rules and external atom guessing rules from Π_{pg}

Replace all $e_{\&g[\vec{y}]}(\vec{x})$ in Π_{pg} by $\&g[\vec{y}](\vec{x})$

return Π_{pg}

Model-building over Acyclic HEX$^{\exists}$-Programs

Definition

A HEX$^{\exists}$*-program* is a finite set of rules of form

$$\forall\vec{X}\exists\vec{Y} : \mathbf{atom}[\vec{X'} \cup \vec{Y}] \leftarrow \mathbf{conj}[\vec{X}], \tag{1}$$

where $\vec{X}$ and $\vec{Y}$ are disjoint sets of variables, $\vec{X'} \subseteq \vec{X}$, $\mathbf{atom}[\vec{X}]$.

Definition

For HEX$^{\exists}$-program Π let $T_{\exists}(\Pi)$ be the HEX-program where each

$$r = \exists\vec{Y} : \mathbf{atom}[\vec{X'} \cup \vec{Y}] \leftarrow \mathbf{conj}[\vec{X}]$$

is replaced by

$$\mathbf{atom}[\vec{X'} \cup \vec{Y}] \leftarrow \mathbf{conj}[\vec{X}], \&exists^{|\tilde{X'}|,|\tilde{Y}|}[r, \vec{X'}](\vec{Y}),$$

where $f_{\&exists^{n,m}}(\mathbf{A}, r, \vec{x}, \vec{y}) = 1$ iff $\vec{y} = \phi_1, \ldots, \phi_m$ is a vector of *fresh and unique null values* for $r, \vec{x}$ and do not appear in Π, and $f_{\&exists^{n,m}}(\mathbf{A}, r, \vec{x}, \vec{y}) = 0$ otherwise.

Model-building over Acyclic HEX$^{\exists}$-Programs

Example

Program Π:

$$\begin{aligned} & employee(john). \quad employee(joe). \\ r_1 : \exists Y : & office(X,Y) \leftarrow employee(X). \\ r_2 : & room(Y) \leftarrow office(X,Y) \end{aligned}$$

Program $T_{\exists}(\Pi)$:

$$\begin{aligned} & employee(john). \quad employee(joe). \\ r_1' : & office(X,Y) \leftarrow employee(X), \&exists^{1,1}[r_1, X](Y). \\ r_2 : & room(Y) \leftarrow office(X,Y) \end{aligned}$$

The unique answer set of $T_{\exists}(\Pi)$ is
$\{employee(john), employee(joe), office(john, \phi_1),$
$office(joe, \phi_2), room(\phi_1), room(\phi_2)\}$.

Model-building over Acyclic HEX$^{\exists}$-Programs

For de-safe programs we do not need the hooks, thus let GroundDESafeHEX be the instantiation of BGroundHEX where

- Repeat repeats exactly once
- Evaluate return always $true$

Then:

Proposition

For de-safe programs Π, $\mathcal{AS}(\mathit{GroundDESafeHEX}(\Pi)) \equiv^{pos} \mathcal{AS}(\Pi_g)$.

Query Answering over Positive HEX$^{\exists}$-Programs

Definition

A $Datalog^{\exists}$-program is a finite set of rules of form $\forall\vec{X}\exists\vec{Y} : \mathbf{atom}[\vec{X}' \cup \vec{Y}] \leftarrow \mathbf{conj}[\vec{X}]$ where $\vec{X}$ and $\vec{Y}$ are disjoint sets of variables, $\vec{X}' \subseteq \vec{X}$.
Disallowed: default negation, general external atoms.

Definition

A homomorphism is a mapping $h : \mathcal{N} \cup \mathcal{V} \rightarrow \mathcal{C} \cup \mathcal{V}$.
A homomorphism h is called substitution if $h(N) = N$ for all $N \in \mathcal{N}$.

Definition

Model of a program: set of atoms M s.t. whenever there is a substitution h with $h(B(r)) \subseteq M$ for some $r \in \Pi$, then $h|_{\vec{X}}(H(r))$ is substitutive to some atom in M.

Definition

A conjunctive query q is of form $\exists\vec{Y} :\leftarrow \mathbf{conj}[\vec{X} \cup \vec{Y}]$ with free variables $\vec{X}$.

Query Answering over Positive HEX$^{\exists}$-Programs

Answer of a CQ q with free variables $\vec{X}$ wrt. model M:

$ans(q, M) = \{h|_{\vec{X}} \mid h$ is a substitution and $h|_{\vec{X}}(q)$ is substitutive to some $a \in M\}$

Answer of a CQ q wrt. a program Π:

$ans(q, \Pi) = \{h \mid h \in ans(q, M)\ \forall M \in mods(\Pi)\}$

Query Answering over Positive HEX$^{\exists}$-Programs

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Definition

Model U of a program Π is universal if, for each $M \in mods(\Pi)$, there is a homomorphism h s.t. $h(U) \subseteq M$.

Proposition

Let U be a universal model of $Datalog^{\exists}$-program Π. Then for each CQ q, $h \in ans(q, \Pi)$ iff $h \in ans(q, U)$ and $h : \mathcal{V} \to \mathcal{C} \setminus \mathcal{N}$.

Query Answering over Positive HEX$^{\exists}$-Programs

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$\Rightarrow$ Key issue: Computing (finite subsets of) a universal model

Query Answering over Positive HEX$^{\exists}$-Programs

Example

Let Π be the following $Datalog^{\exists}$-program:

$$person(john). \quad person(joe).$$
$$r_1 : \exists Y : father(X,Y) \leftarrow person(X).$$
$$r_2 : \quad person(Y) \leftarrow father(X,Y).$$

Then $T_{\exists}(\Pi)$ is the following program:

$$person(john). \quad person(joe).$$
$$r_1' : \quad father(X,Y) \leftarrow person(X), \&exists^{1,1}[r_1, X](Y).$$
$$r_2 : \quad person(Y) \leftarrow father(X,Y).$$

Query Answering over Positive HEX$^{\exists}$-Programs

Input: A HEX-program $\Pi = T_{\exists}(\Pi_{\exists})$ for some $Datalog^{\exists}$-program $\Pi_{\exists}$, the count of freeze steps c_{freeze}
Output: A ground HEX-program Π_g s.t. $\mathbf{A} \in \mathcal{AS}(\Pi_g)$ is sound and complete for query answering

$\Pi_p = \Pi \cup \{r_{inp}^{\&g[\vec{Y}](\vec{X})} \mid \&g[\vec{Y}](\vec{X}) \text{ in } r \in \Pi\}$

Replace all external atoms $\&g[\vec{Y}](\vec{X})$ in all rules r in Π_p by $e_{r,\&g\vec{Y}}(\vec{X})$

for $f = 0, \ldots, c_{freeze}$ **do**
  $PIT \leftarrow \emptyset$
  $NewInputTuples \leftarrow \emptyset$
  repeat
    $\Pi_{pg} \leftarrow \text{GroundASP}(\Pi_p)$
    for $\&g[\vec{Y}](\vec{X})$ *in a rule* $r \in \Pi$ **do**
      $\mathbf{A}_{ma} = \{\mathbf{T}p(\vec{c}) \mid a(\vec{c}) \in A(\Pi_{pg}), p \in \vec{Y}_m\} \cup \{\mathbf{F}p(\vec{c}) \mid a(\vec{c}) \in A(\Pi_{pg}), p \in \vec{Y}_a\}$
      for $\mathbf{A}_{nm} \subseteq \{\mathbf{T}p(\vec{c}), \mathbf{F}p(\vec{c}) \mid p(\vec{c}) \in A(\Pi_{pg}), p \in \vec{Y}_n\}$ *s.t.* $\nexists a : \mathbf{T}a, \mathbf{F}a \in \mathbf{A}_{nm}$ **do**
        $\mathbf{A} = (\mathbf{A}_{ma} \cup \mathbf{A}_{nm} \cup \{\mathbf{T}a \mid a \leftarrow\in \Pi_{pg}\}) \setminus \{\mathbf{F}a \mid a \leftarrow\in \Pi_{pg}\}$
        for $\vec{y} \in \{\vec{c} \mid r_{inp}^{\&g[\vec{Y}](\vec{X})}(\vec{c}) \in A(\Pi_{pg})$ *which is not homomorphic to any* $a \in PIT\}$ **do**
          Let $O = \{\vec{x} \mid f_{\&g}(\mathbf{A}, \vec{y}, \vec{x}) = 1\}$
          $\Pi_p \leftarrow \Pi_p \cup \{e_{r,\&g[\vec{y}]}(\vec{x}) \vee ne_{r,\&g[\vec{y}]}(\vec{x}) \leftarrow\mid \vec{x} \in O\}$
          $NewInputTuples \leftarrow NewInputTuples \cup \{r_{inp}^{\&g[\vec{Y}](\vec{X})}(\vec{y})\}$
    $PIT \leftarrow PIT \cup NewInputTuples$
  until Π_{pg} *did not change*

Remove input auxiliary rules and external atom guessing rules from Π_{pg}
Replace all $e_{\&g[\vec{y}]}(\vec{x})$ in Π_{pg} by $\&g[\vec{y}](\vec{x})$
return Π_{pg}

Query Answering over Positive HEX$^{\exists}$-Programs

Example (ctd.)

Let Π be the following $Datalog^{\exists}$-program:

$$person(john). \quad person(joe).$$
$$r_1 : \exists Y : father(X,Y) \leftarrow person(X).$$
$$r_2 : \quad person(Y) \leftarrow father(X,Y).$$

Then $T_{\exists}(\Pi)$ is the following program:

$$person(john). \quad person(joe).$$
$$r_1' : \quad father(X,Y) \leftarrow person(X), \&exists^{1,1}[r_1, X](Y).$$
$$r_2 : \quad person(Y) \leftarrow father(X,Y).$$

For $c_{freeze} = 1 \Rightarrow$ program with single answer set
$\{person(john), person(joe), father(john, \phi_1), father(joe, \phi_2), person(\phi_1), person(\phi_2)\}$

Query Answering over Positive HEX$^{\exists}$-Programs

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For $c_{freeze} = 1 \Rightarrow$ program with single answer set
$\{person(john), person(joe), father(john, \phi_1), father(joe, \phi_2), person(\phi_1), person(\phi_2)\}$

Proposition

For a shy program Π, GroundDatalog$^{\exists}(\Pi, k)$ has a unique answer set which is sound and complete for answering CQs with up to k existential variables.

Function Symbols

Definition (Terms)

The set of terms $\mathcal{T}$ is defined as the least set s.t. $\mathcal{T} \supseteq \mathcal{V} \cup \mathcal{C}$ and $f \in \mathcal{C}, t_1, \ldots, t_n \in \mathcal{T}$ implies $f(t_1, \ldots, t_n) \in \mathcal{T}$.

For each $k \in \mathbb{N}$ two external predicates $\&compose_k$ and $\&decompose_k$ with $ar_{\mathsf{I}}(\&compose_k) = 1 + k$ and $ar_{\mathsf{O}}(\&compose_k) = 1$ and $ar_{\mathsf{I}}(\&decompose_k) = 1$ and $ar_{\mathsf{O}}(\&decompose_k) = 1 + k$.

Following [Calimeri et al., 2007],

$$f_{\&compose_k}(\mathbf{A}, f, X_1, \ldots, X_k, T) = f_{\&decompose_k}(\mathbf{A}, T, f, X_1, \ldots, X_k) = 1,$$

iff $T = f(X_1, \ldots, X_k)$.

Note: $\&decompose_k$ supports a well-ordering

Function Symbols

Definition

Let Π be a HEX-program with function symbols. Then $T_f(\Pi)$ is the program where each $f(t_1, \ldots, t_n)$ in a rule r is recursively replaced by a new variable V.
If $f(t_1, \ldots, t_n)$ appears in $H(r)$ or in the input list of some external atom in $B(r)$, then $\&compose_n[f, t_1, \ldots, t_n](V)$ is added to $B(r)$, and otherwise $\&decompose_n[V](f, t_1, \ldots, t_n)$ is added to $B(r)$.

Example

Program Π:

$$q(z).\ q(y).$$
$$p(f(f(X))) \leftarrow q(X).$$
$$r(X) \leftarrow p(X).$$
$$r(X) \leftarrow r(f(X)).$$

Then $T_f(\Pi)$ is:

$$q(z).\ q(y).$$
$$p(V) \leftarrow q(X), \&compose_1[f, X](U), \&compose_1[f, U](V).$$
$$r(X) \leftarrow p(X).$$
$$r(X) \leftarrow r(V), \&decompose_1[V](f, X).$$

Conclusion

ASP Programs with External Sources

- Ordinary safety not sufficient due to value invention
- Notion of liberal domain-expansion safety guarantees finite groundability

Contribution

- Domain-specific existential quantifier in heads realized by external sources
  Advantage: Easy extensibility, e.g., data types, side constraints
- Grounding algorithm extended by application-specific termination hooks
- Instances: model building over acyclic programs, query answering with logical existential quantifier, function symbols

Future Work

- Combination of query answering with function symbols, default negation
- Model-building over programs with infinite but finitely representable models

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